Pre-class Warm-up!!!

Question: True or False, for vectors v_1, \ldots, v_6 in R^n ?

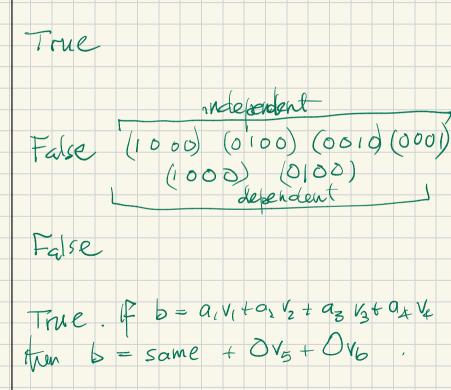
a. If v_1, \ldots, v_6 are linearly independent then v_1, \ldots, v_4 are necessarily linearly independent.

b. If v_1, \ldots, v_4 are lin. indep. then v_1, \ldots, v_6 are necessarily lin. indep.

c. If v_1, \ldots, v_6 span R^n then v_1, \ldots, v_4 necessarily span R^n.

d. If v_1, \ldots, v_4 span R^n then v_1, \ldots, v_6 necessarily span R^n.

True False



Section 4.4: Bases and dimension

We learn:

- The meaning of the word basis, and a broader definition of the word dimension.
- Theorem: Any two bases for a vector space have the same size.
- Theorem: A basis is a maximal independent set, and also a minimal spanning set.
- How to find a basis for various vector spaces: the solution set to a homogeneous system of equations, lines and planes in R^3 .

Definition on page 235: A set of vectors $S = \{v_1, \dots, v_k\}$ is a basis for a vector space V if and only if a. the vectors are independent, and b. they span V. Example. (1, 0, -1), (1, 2, -2), (1, 6, -3) We have already seen these are linearly independent and spon R (Reduce oz 6 to reduced echelon $form \begin{bmatrix} i & b & o \\ o & i & o \end{bmatrix} \cdot e(c)$ These vectors are a bains for TR3

Equivalently: S is a basis for V <=> every vector \$ in V can be written) uniquely as a linear combination of the Vectors in S, If unque, then the O rector is unquele a linear combination of vectors in S so they are independent. $|f \alpha_1 \vee (+ \cdot \cdot + \alpha_k \vee_k) = |V_1 + \cdot \cdot + D_k \vee_k = W$ then $(a_1 - b_1)v_1 + - (a_2 - b_2)v_2 = W - w = O$ Example: the standard basis for R^3 \mathcal{S} [], [], [], It is a paris independent => ai-bi= 0 always

Like question 12 - 14: Find a basis for the plane in R^3 with equation 2x - y + 3z = 0.

Solution: The plane is the solution set to
$$\begin{bmatrix} 2 & -1 & 3 \end{bmatrix} \begin{bmatrix} 1 & 1 \\ 2 & 2 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ -1 \end{bmatrix} \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix} = \begin{bmatrix} 2 & 3 \\ 2 & 3 \end{bmatrix}$$

$$\begin{bmatrix} 2 & -1 & 3 \end{bmatrix} \text{ is in cehelon form. } y, z \text{ are}$$

free variables. The general solution
is
$$\begin{bmatrix} X \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} (y - 3z) \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{2} \\ y \\ z \end{bmatrix} = \begin{bmatrix} \frac{1}{2}$$

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Span / Are they independent?
If
$$y \begin{bmatrix} 1/2 \\ 1/2 \\ 0 \end{bmatrix} + 2 \begin{bmatrix} -3/2 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} y \\ 2 \\ 2 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$
 then

y = 0 and z=0, so they are independent,

Like questions 15 - 26: Find a basis for the solution space of the system

$$w + 2x + 3y + 4z = 0$$

$$2w + 4x + 7y + 9z = 0$$

$$3w + 6x + 9y + 12z = 0$$

Ouestion: determine whether the vectors (1, 0, -1), (1, 2, -2), (1, 6, -3) form a basis for R^3 .

Solution: Reduce (0 2 6 -1 -2 -3

The rectors are independent and span (suy more) as before.

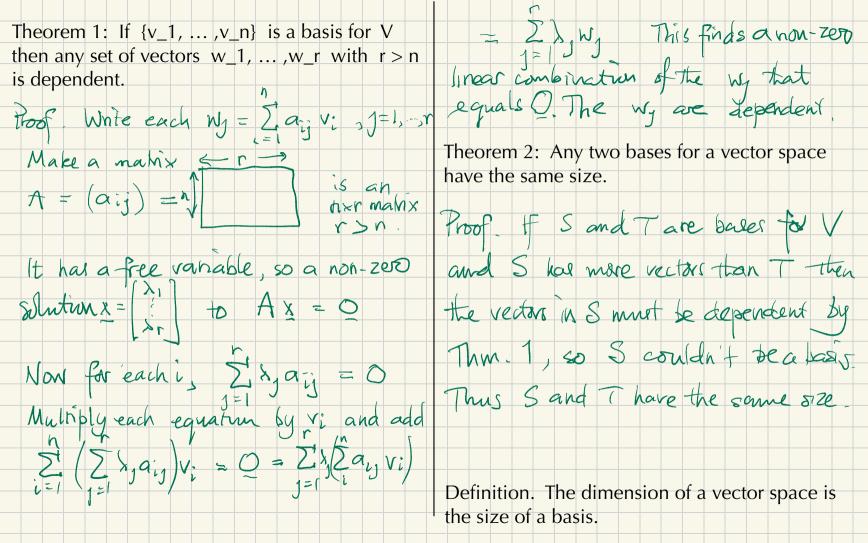
to

Further question:

a. Does the vector (1,6,-3) lie in the span of the vectors (1, 0, -1) and (1, 2, -2)?

b. Does the vector (1,2,-2) lie in the span of the vectors (1, 0, -1) and (1, 6 -3)?

Yes No



Pre-class Warm-up!!!

Question: have we already proved the following theorem?:

Theorem

Let V be a vector space of dimension n and let S be a set of n vectors in V. If S is linearly independent then S spans V and hence is a basis for V.

Yes No

What about:

Theorem

Let S be a set of n vectors in R^n . If S is linearly independent then S spans V and hence is a basis for V.

Rn



Theorem 3 (second 1/2 of it) Theorem 3 (first 1/2 of it) (c) If S is a set of linearly independent vectors in Let V be a vector space of dimension n and a vector space V then S is contained in a basis let S be a set of n vectors in V. for V. S can be extended to a basis for V. (a) If S is linearly independent then S is a basis for V. (d) If S is a set of vectors that spans V then S (b) If S spans V then S is a basis for V. contains a basis for V. Proof (a), Sis lin, ind, so is contained (c) is in the homework (questions 29 and 30). (d) is questions 31 and 32. in a basis by (c). This basis has n vectors (c) implied that every vector space has a basis. S is the basis (d). Assume S is a Finile set of vectors (b) Smitar, If they are indexndent, they are a basis, done. Otherwise come vector can be contreh as a linear combination of the others. Remove it. The span of the smaller set is also V. Repeat until we get an independent set; a basis!

Example (like example 4):

Let V be the set of polynomials

 $a_0 + a_1 x + a_2 x^2 + a_3 x^3$

(a) Show that V has dimension 4.
(b) Show that 1, 1+x, x+x^2, x^2+x^3 is a basis for V